separations b. Other parameters are the same as in Fig. 2. The curves have been obtained from the far-field angular intensity distribution

$$|\phi(r,\vartheta)|^2 \simeq |a_1|^2 \frac{1}{2\pi kr} \frac{k^2\cos^2\vartheta}{[\Im m(\beta_1)]^2 + [\Re e(\beta_1) - k\sin\vartheta]^2} \tag{10}$$

with the angular divergence Δ defined as the half-width of this distribution. Here, ϑ is an angle between the vector $\boldsymbol{r} = (x, z)$ and the positive x-axis, and $r = |\boldsymbol{r}|$. Equation (10) has been calculated with the help of a appropriate two-dimensional (2-D) diffraction integral [7].

It is seen from Fig. 5 that the value of Δ decreases and becomes a smoother function of d when the grating separation becomes larger. This is a consequence of a weaker coupling of the guided mode with the grating at large separations and smaller changes of the propagation parameters with the varying grating period. Smoothing of the Δ-curves goes together with the narrowing of the radiated beams. It is a fortunate property for applications, where the wide scanning angles and narrow beams are of great importance.

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The Influence of Ground-Plane Width on the Ohmic Losses of Coplanar Waveguides with Finite Lateral Ground Planes

Giovanni Ghione and Michele Goano

***Abstract*— In this paper, analytical computer-aided-design (CAD)-oriented conformal-mapping approximations are presented for the high-frequency attenuation of symmetric and asymmetric coplanar waveguides (ACPW's) with finite-extent lateral ground planes. A discussion is presented on the effect of ground-plane width on the losses, and design criteria are derived.**

***Index Terms*—Attenuation, conformal mapping, coplanar waveguides, design automation software.**

I. INTRODUCTION

Coplanar waveguides (CPW's) are currently used extensively in both microwave integrated circuits (MIC's) and electro-optic components on $LiNbO_3$ substrates. In practice, such lines always have ground planes of finite width, as shown in the insets of Figs. 1 and 2 for the symmetric CPW and the asymmetric CPW, respectively. While in the design of MIC's the choice of lateral ground width is mainly driven by layout considerations (i.e., the ground-plane width $c - b$ should be large enough to avoid coupling between neighboring lines, without unnecessarily increasing the circuit size [1]), the performance optimization of electro-optic components such as amplitude and phase modulators often requires the use of very narrow lateral ground planes, as discussed in [2], [3].

From the standpoint of the CPW performances, reducing the ground-plane width causes an increase of the line impedance [see [1] for the symmetric and [4] for the asymmetric case, which can be derived from the analysis of the asymmetric coplanar stripline (ACPS)] but also of the line losses, which can significantly exceed those of the ideal structure if ground planes are narrow. To analyze such an effect, this paper presents a new closed-form expression for the skin-effect conductor attenuation of the symmetric CPW with finite-extent lateral ground planes, while the losses of the asymmetric CPW are derived by suitably rearranging the expression valid for the ACPS [4].

The analysis technique is the conformal mapping method introduced by Owyang and Wu for the analysis of conductor losses in the symmetric CPW with infinite lateral ground planes [5], and later exploited by Ghione [4] for the loss analysis of general asymmetric CPW's and striplines. The analytical expressions derived are compared with numerical results obtained from two electromagnetic simulators (HFSS[1] and Explorer[2]) with fairly good agreement. Finally, some design criteria are derived both for the symmetric and asymmetric case.

II. ANALYSIS

Despite its well-known limitations in the low-frequency range [6], [7], the skin-effect analysis of losses in a planar transmission line

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[1] Hewlett-Packard Company, Santa Rosa, CA, *HP 85180A High-Frequency Structure Simulator. User's Reference*, May 1992.
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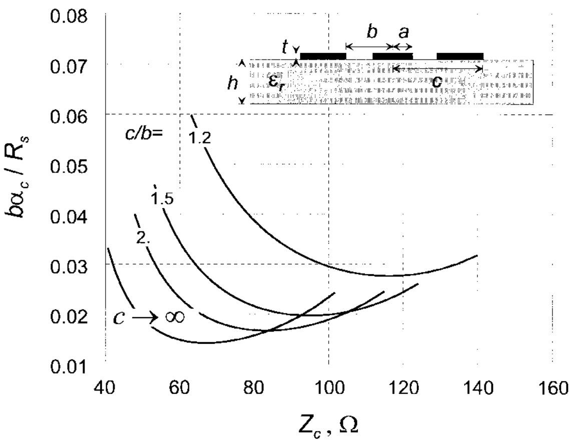

Fig. 1. Normalized attenuation for symmetric CPW (see inset) with finite ground-plane width versus the line impedance for several values of the normalized ground-plane width c. The strip thickness is $t/b = 0.01$ and the GaAs substrate ($\epsilon_r = 13$) is thick ($h/b \gg 1$).

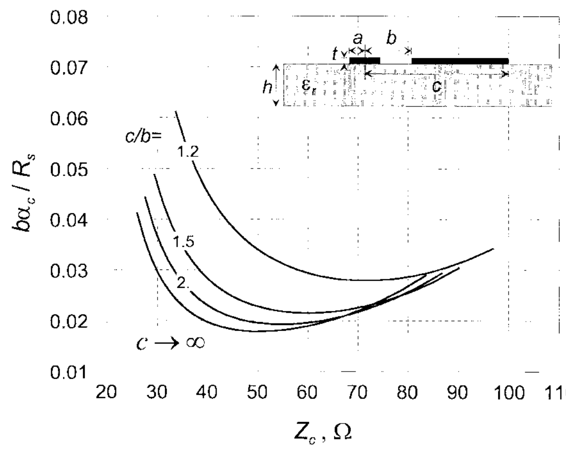

Fig. 2. Normalized attenuation for asymmetric CPW (see inset) with finite ground-plane width versus the line impedance for several values of the normalized ground-plane width c. The strip thickness is $t/b = 0.01$ and the GaAs substrate ($\epsilon_r = 13$) is thick ($h/b \gg 1$).

based on the high-frequency current distribution can still be assumed as the basis to investigate the effect of the geometry on the attenuation of thin lines (i.e., such that the line thickness is much smaller than the strip width $2a$, the slot width $b-a$, and the ground-plane width $c-b$). Moreover, by introducing an equivalent line thickness, skin-effect formulas can be readily exploited so as to generate approximations able to also cover the low-frequency range, as discussed in [8].

The skin-effect loss analysis is based on the well-known expression of the per-unit-length conductor loss attenuation α_c:

$$\alpha_c = \frac{R_s}{2Z_c I^2} \oint |J|^2 \, dl \tag{1}$$

where R_s is the surface resistance, Z_c is the line impedance, I is the total current carried by the line, J is the current density, and the line integral is defined on the conductor periphery.

A. Symmetric CPW

The high-frequency current density of the symmetric CPW with finite-extent ground planes can be approximated through the static charge distribution of the line, which in its turn is estimated by means of conformal mapping according to the technique in [4]. Some modifications are implemented with the aim to exploit the line symmetry. The square of the current density can be integrated on the line periphery in closed form if the line thickness is much smaller than the strip, slot, and ground-plane widths, as discussed in [4]. After straightforward but lengthy analytical manipulations, one finally obtains the following result for the symmetric CPW attenuation:

$$\begin{aligned}\alpha_c^{\mathrm{CPW}} =& \frac{R_s\sqrt{\epsilon_{\mathrm{eff}}^{\mathrm{CPW}}}}{480\pi K(k_1)K(k_1')(1-k_{ab}^2)} \\ &\times \left\{ \frac{1}{a}\left[\log\left(\frac{8\pi a}{t}\,\frac{1-k_{ab}}{1+k_{ab}}\,\frac{1+k_{ac}}{1-k_{ac}}\right)+\pi\right] \right. \\ &+ \frac{1}{b}\left[\log\left(\frac{8\pi b}{t}\,\frac{1-k_{ab}}{1+k_{ab}}\,\frac{1-k_{bc}}{1+k_{bc}}\right)+\pi\right]\frac{1-k_{ac}^2}{1-k_{bc}^2} \\ &+ \left. \frac{1}{c}\left[\log\left(\frac{8\pi c}{t}\,\frac{1+k_{ac}}{1-k_{ac}}\,\frac{1-k_{bc}}{1+k_{bc}}\right)+\pi\right]\cdot\frac{1-k_{ab}^2}{1-k_{bc}^2}k_{bc}^2 \right\}\end{aligned} \tag{2}$$

where K is the complete elliptic integral of the first kind, and

$$\begin{aligned} k_1 &= \sqrt{\frac{1-k_{ab}^2}{1-k_{ac}^2}} \\ k_{ab} &= a/b \\ k_{bc} &= b/c \\ k_{ac} &= a/c. \end{aligned}$$

Analytical approximations to the effective permittivity $\epsilon_{\mathrm{eff}}^{\mathrm{CPW}}$ and characteristic impedance Z_c^{CPW} of the line have been presented in [1, eq. (17a), (17b), (18)]. For $c \to \infty$, one has $k_{ac} \to 0$, $k_{bc} \to 0$, and (2) yields, in the limit, the expression for the attenuation of the standard CPW with infinite lateral ground planes [4, eq. (45)].

B. Asymmetric CPW

The asymmetric CPW (ACPW) with finite-extent ground plane coincides with the asymmetric coplanar strip line (CPS) discussed in [4]. The characteristic parameters and attenuation of this structure can be derived from the expressions in [4] by performing a change of notation. The attenuation of the ACPW with finite ground plane reads [4, eq. (54)]

$$\begin{aligned}\alpha_c^{\mathrm{ACPW}} =& \frac{R_s\sqrt{\epsilon_{\mathrm{eff}}^{\mathrm{ACPW}}}}{480\pi K(k_2)K(k_2')} \\ &\times \left\{ \left[\log\left(\frac{8\pi a}{t}k_2\right)+\pi\right]\frac{1}{2a} \right. \\ &+ \left[\log\left(\frac{4\pi(c-b)}{t}k_2\right)+\pi\right]\frac{1}{c-b} \\ &+ \left[\log\left(\frac{4\pi(b-a)}{t}k_2'\right)+\pi\right]\frac{1}{b-a} \\ &- \left. \left[\log\left(\frac{4\pi(a+c)}{t}k_2'\right)+\pi\right]\frac{1}{a+c} \right\}\end{aligned} \tag{3}$$

where

$$k_2 = \sqrt{\frac{1-k_{ab}}{1+k_{ab}}\,\frac{1+k_{ac}}{1-k_{ac}}}.$$

The effective permittivity $\epsilon_{\mathrm{eff}}^{\mathrm{ACPW}}$ and the characteristic impedance Z_c^{ACPW} may be obtained by replacing $k \to k_2, b_1 - a \to a, b_1 + a \to b$ and $b_1 + b_2 \to c$ in [4, eq. (12), (55)].

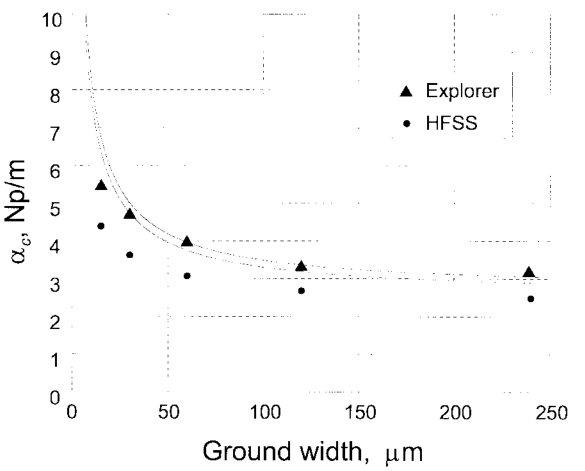

Fig. 3. Attenuation of asymmetric CPW on a thick GaAs substrate versus ground-plane width as computed from (2) (continuous line) and including the correction in [8] (dashed line). Dots and triangles are the results obtained from the HFSS[1] and Explorer[2] electromagnetic simulators. The line width is $2a = 140$ μm and the ground-plane spacing is $b - a = 30$ μm; the line thickness is 5 μm and the frequency is 5 GHz.

III. Results and Discussion

The conductor attenuation for a symmetric and asymmetric CPW with finite ground planes on a thick GaAs ($\epsilon_r = 13$) substrate was evaluated for several values of the line aspect ratio a/b, taking as a parameter the ratio c/b. The normalized attenuation $b\alpha_c/R_s$ is plotted versus the line impedance in Figs. 1 and 2 for the symmetric case and asymmetric case, respectively; the normalized line thickness is $t/b = 0.01$. Since in this case $\epsilon_{\text{eff}} \approx (\epsilon_r + 1)/2$, the behavior shown holds for arbitrary substrate permittivity, provided that the impedance and attenuation are suitably rescaled.

As expected, if all the line dimensions are kept constant and the ground-plane width c is decreased, the attenuation always increases. However, if the comparison is made at constant line impedance, the decrease of the ground-plane width increases losses only for low and medium impedance lines, while this trend can be reversed for high impedance lines. The decrease in the ground-plane width also causes an increase of the optimum impedance for minimum loss. However, the influence of ground-plane width is more pronounced in the asymmetric case than in the symmetric one. In fact, one can notice that while for large ground-plane widths, the attenuation of the ACPW is *lower* than the one of the CPW—this behavior is reversed for extremely small ground-plane widths. The slightly lower attenuation exhibited by the ACPW can be explained by considering that high-frequency losses are dominated by edge effects. In the ACPW, only one closely coupled edge is present, while in the CPW, the added effect of the second closely coupled edge is not compensated for by the decrease in the ground-plane losses caused by the increase of the ground-plane periphery.

Approximate design criteria can be derived from the analysis formulas presented. In particular, for arbitrary substrates (thickness and permittivity) one obtains the following.

- For the symmetric CPW, the increase in attenuation with respect to the ideal case ($c = \infty$) is less than 10% if $c > 2b$, for $0.05 < a/b < 0.95$.
- For the asymmetric CPW, the increase in attenuation with respect to the ideal case ($c = \infty$) is less than 10% if $c > 3.5b$, for $0.05 < a/b < 0.95$.

In order to validate the expression proposed, the attenuation of several symmetric and asymmetric lines was computed as a function of the ground-plane width by means of two state-of-the-art electromagnetic simulators, HFSS[1] and Explorer,[2] and compared with the present approach. The corrections in [8] were also implemented for the sake of comparison.

The behavior shown in Fig. 3 is typical for all cases considered. As expected, the correction in [8] is not overly significant in the skin-effect regime, and leads to a very small decrease in the attenuation. The attenuation as evaluated by HFSS[3] through its 2-D Wave Module was consistently found to be slightly lower than the one provided by the present approach; however, convergence studies reveal a little increase of the HFSS result with increasing port field accuracy and denser discretization mesh. Since the accuracy criterion exploited by HFSS is based on the self-consistency of the field distribution, the convergence in the attenuation turns out to be very slow, so that computational limitations do not practically enable one to ascertain whether the discrepancy (which has been observed also by other investigators [9]) is physical, or rather a numerical artifact. Concerning Explorer, the agreement turns out to be good, although some care must be exerted, since this simulator sometimes yields anomalous results with respect to frequency.

IV. Conclusions

Analytical expressions for the high-frequency conductor losses of symmetric and asymmetric CPW's with lateral ground planes of finite width have been presented. The decrease of the ground-plane width causes an increase of the conductor attenuation and a shift of the optimum impedance for minimum losses toward higher values. The effect of finite-width ground planes is shown to be negligible on the line losses (less than 10% increase with respect to the ideal case) if the conditions $c > 2b$ (symmetric case) and $c > 3.5b$ (asymmetric case) are met.

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